Stereographically Projected Cosmological Simulations

Cosmological zoom-in simulations with stereographic projection

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Outline

1. Introduction
   - Concordance Cosmology
   - The Large Scale Structure
2. SPCS
   - Zoom-in simulations
   - The Algorithm and Results

Einstein equations for the universe

$$R_{\mu\nu} - \tfrac{1}{2} R \, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Cosmological principle:
Friedmann–Lemaître–Robertson–Walker metric

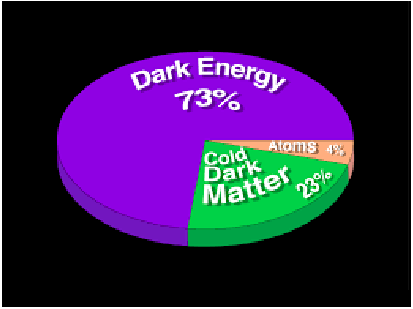
$$ds^2 = -c^2 \mathrm{d}t^2 + a(t)^2 \left[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2 \right) \right]$$

Friedmann equations

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right)$$

The ingredients

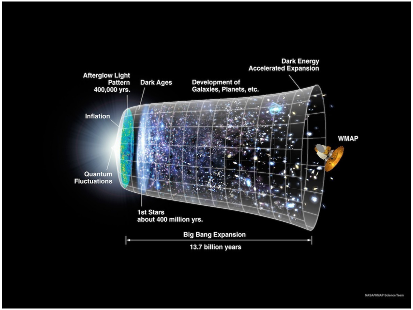


- Ordinary matter (baryons, we know them)
- Dark Matter (no detection, some good ideas)
- Dark Energy (cosmological constant)

Concordance Model

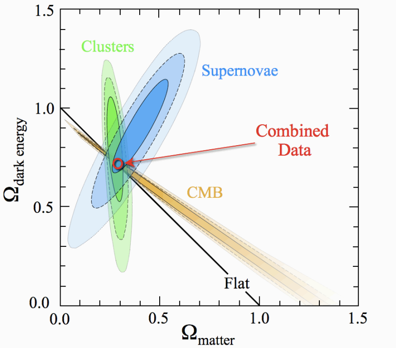
The standard cosmological model: ΛCDM

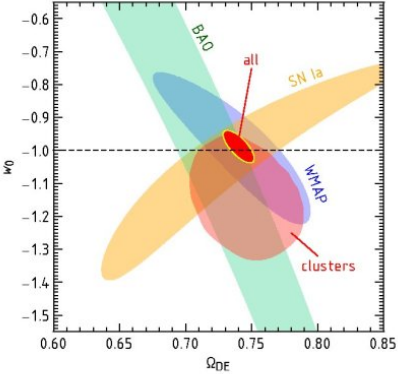
- Cosmological constant (or Dark Energy) (Λ)
- Cold Dark Matter (CDM)
- 6 independent parameter
- Fits well with the measurements



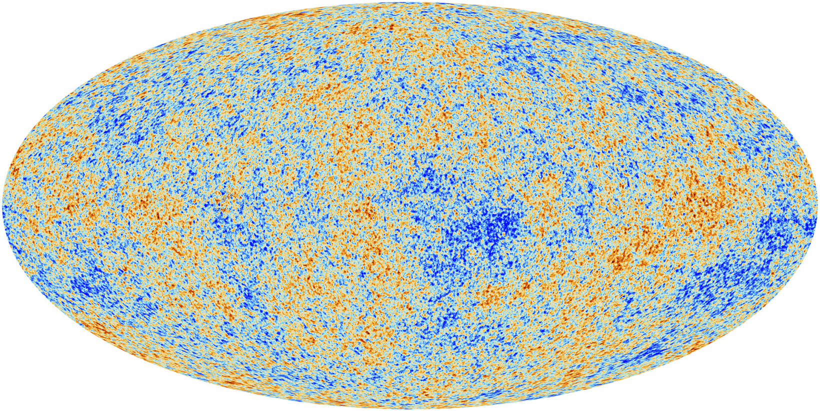
Concordance Model

BAO+lensing+CMB+clusters+redshift distortions, etc.



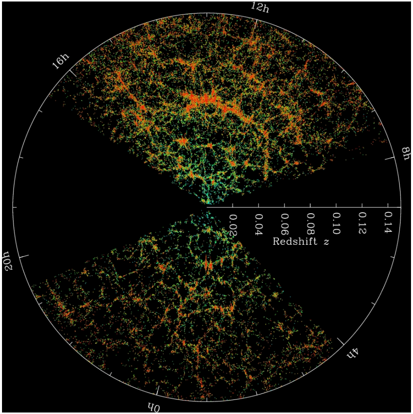


Cosmic Microwave Background



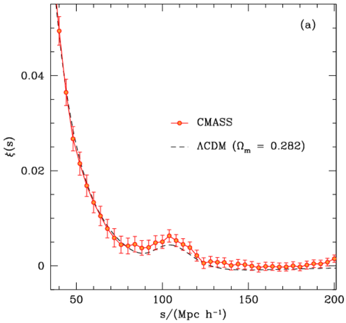
Cosmic microwave background seen by Planck
Small, $\rho/\rho_0 \simeq 10^{-5}$ fluctuations in the density field

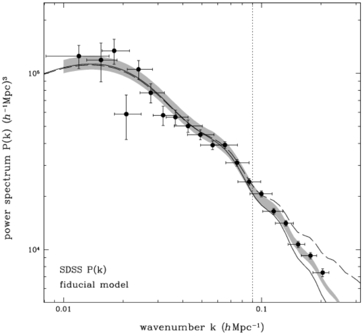
Large Scale Structure



Large fluctuations in the galaxy distribution. Density field: $\rho/\rho_0 \simeq 10^6$

Statistics of the density distribution





Sanchez, Ariel G. et al. Mon.Not.Roy.Astron.Soc. 425 (2012) 415 arXiv:1203.6616 [astro-ph.CO]

Yoo, Jaiyul et al. Astrophys.J. 698 (2009) 967-985 arXiv:0808.2988 [astro-ph]

How to calculate the matter distribution as a function of time?

These structures formed by gravitational instability
Initial conditions: CMB power spectra.
At early times:

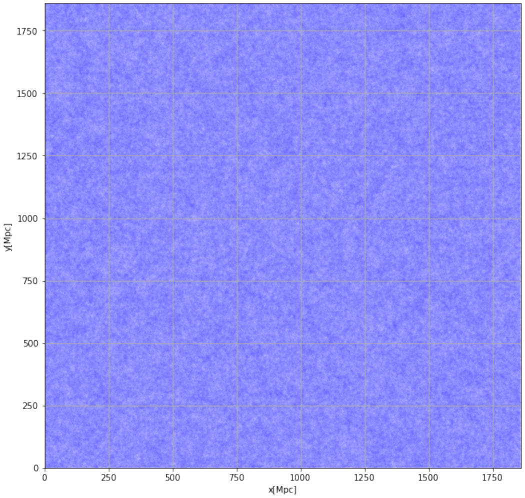
- The fluctuations are small
- Linear structure formation
- Perturbation theory can be used

Late times:

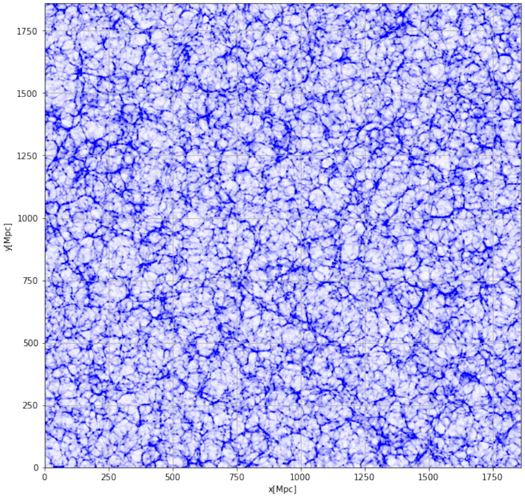
- Non-linear structure formation
- Linear theory cannot be used at small redshifts
- Cosmological simulations

Cosmological simulations

- N-body simulations
- Dark and baryonic matter
- Newtonian gravity in an expanding periodic box
- Number of particles are very large: $N \sim 10^6 - 10^{12}$
- IC: from CMB and perturbation theory



ΛCDM, $z = 19$, $L_{box} = 1860.1 Mpc$, $N = 3.43 \cdot 10^8$



ΛCDM, $z = 0.0$, $L_{box} = 1860.1 Mpc$, $N = 3.43 \cdot 10^8$

Difficulties with cosmological simulations

1. High-performance computing needs for the force calculation
2. For a very fine mass resolution (eg. for galaxy formation simulations), the number of particles can be very large (high memory requirement)
3. Directions in the simulation box are not equivalent (periodic box)
4. Measurements are done on the sky (spherical)

If we only intrested in small scales, then the soultion for 1 and 2: Zoom-in simulations

Zoom-in simulations

Idea: Run simulations with different spatial resolutions
Two main step:

1. The parent simulation is run with low mass resolution and with large simulation volume, to minimize the effect of the periodic boundaries.
2. After the parent simulation is finished, and the small "volume of interest" is chosen, a second simulation is run with spatially varying resolution with the same simulation volume.

There are only few nested cubic volumes with different resolutions and same centre in this case.

Our new method: Stereographically Projected Cosmological Simulations

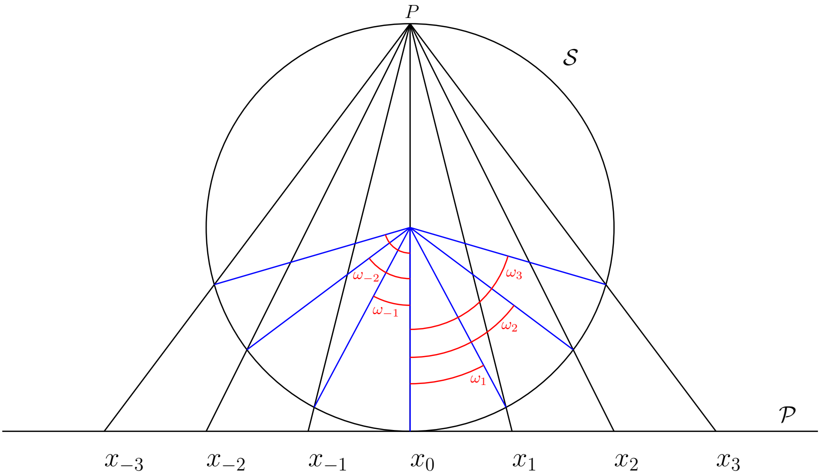
The stereographic projection is a well known bijective geometrical transformation that projects a sphere ($\mathcal{S}$) onto a plane ($\mathcal{P}$). In the one dimensional case:

$$x_i = d_s \cdot \tan\left(\frac{\omega_i}{2}\right)$$

inverse transformation:

$$\omega_i = 2 \cdot \arctan\left(\frac{x_i}{d_s}\right)$$

One dimensional case



Three dimensional case

The easiest way to do the stereographic projection in three dimension, is to use spherical coordinate system. In this case, only the radial coordinate is transformed. If ω_i, ϑ_i, and φ_i are the coordinates in the surface of the four dimensional sphere, then:

$$r_i = d_s \cdot \tan\left(\frac{\omega_i}{2}\right)$$

the inverse transformation:

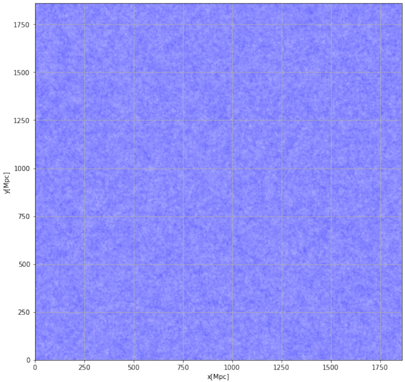
$$\omega_i = 2 \cdot \arctan\left(\frac{r_i}{d_s}\right)$$

We use these transformations to generate a spherical initial condition for our code.

Initial Conditions

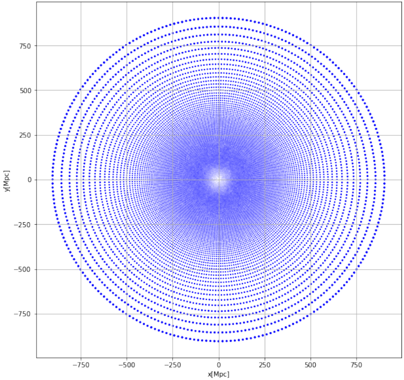
- Input: Periodic initial conditions from NgenIC or 2LPTic
- Choose a Volume of Interest (VOI)
- We use a large spherical region from the initial volume ($D_{sphere} = L_{box}$)
- We transform this space with inverse 3D stereographic projection
- Our code bins the 4 dimensional spherical surface with using HEALPix in ϑ and φ coordinates, and with a simple equivalent size binning in ω
- The particles are united in each bin
- After this, the code uses the stereographic projection to transform the coordinates of these particles back to cartesian coordinate system.

Initial Conditions



Input 2LPTic initial condition

Initial Conditions

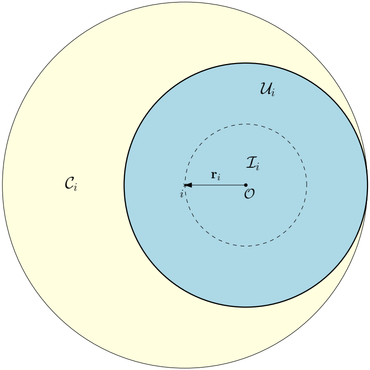


As a result, we get a spherical initial condition with constant angular and decreasing radial resolution in radial direction.

N-body simulations

- gravitation only
- non-periodic boundary conditions
- expansion rate: Friedmann equations

The forces from the boundary conditions can be calculated by the shell theorem.
We assume that the density field is always homgeneous outside.



Homogeneous case:

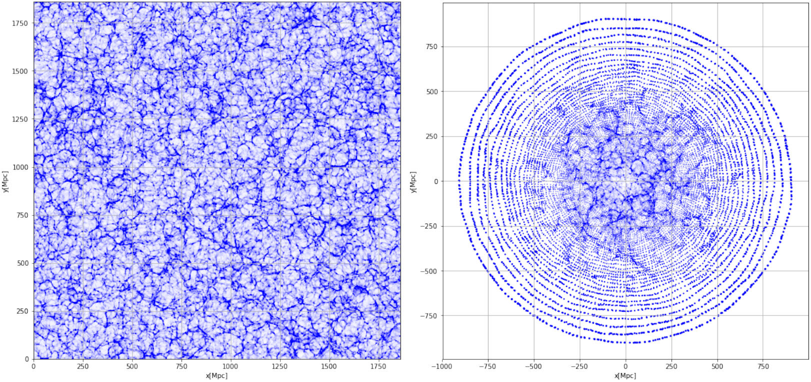
$$0 = \mathbf{F}_{\mathcal{C}'} + \mathbf{F}_{\mathcal{U}_i} + \mathbf{F}_{\mathcal{I}_i}$$

$$\mathbf{F}_{\mathcal{C}_i} = -\mathbf{F}_{\mathcal{I}_i} = G m_i 4\pi \frac{\mathbf{x}_i}{|\mathbf{x}_i|} \int_0^{r_i} r^2 \overline{\rho} dr = \frac{4\pi G m_i}{3} \overline{\rho} \mathbf{x}_i$$

The SPCS code

- Cosmological N-body code
- DM only
- $\sim N^2$ force calculation
- Periodic and spherical simulations
- Parallelized with OpenMP and CUDA

Results


y[Mpc]
x[Mpc]
0
250
500
750
1000
1250
1500
1750
y[Mpc]
x[Mpc]
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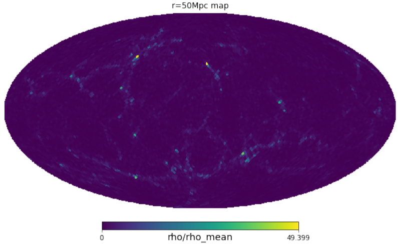
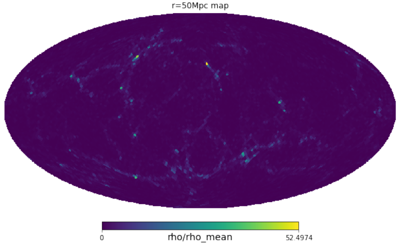
−750
−500
−250
0
250
500
750


r=50Mpc map
0
rho/rho_mean
49.399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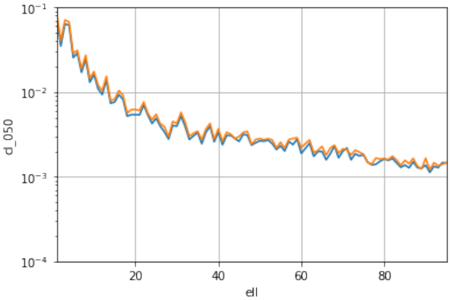
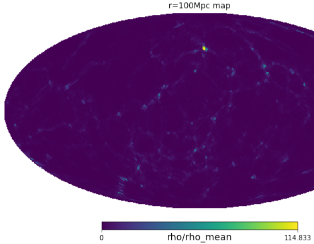

10^{-1}
10^{-2}
10^{-3}
10^{-4}
d_050
20
40
60
80
ell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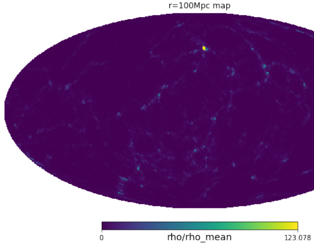
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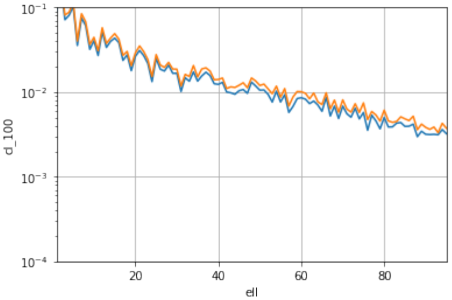
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10^{-2}
10^{-3}
10^{-4}
d_100
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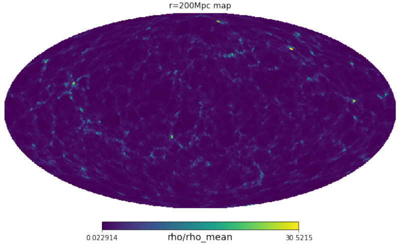
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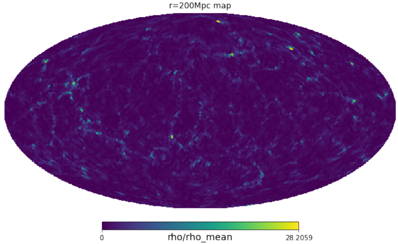
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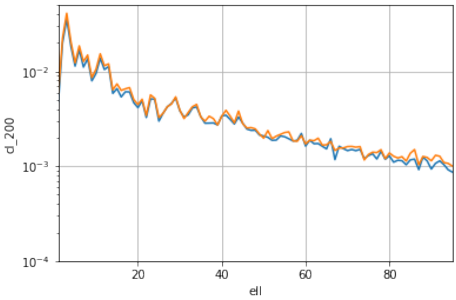
d_200
10^{-2}
10^{-3}
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20
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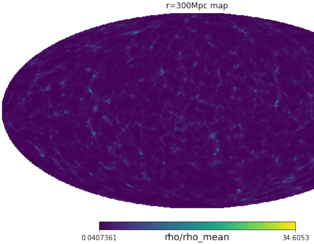
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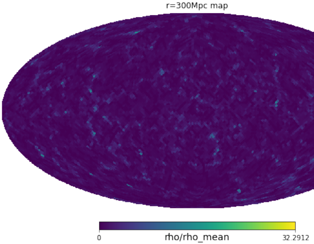
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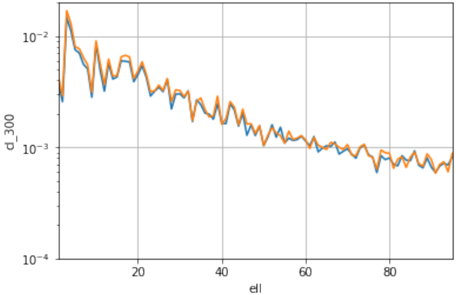
d_300
10^{-2}
10^{-3}
10^{-4}
20
40
60
80
ell

Summary

- SPCS can reproduce mass distribution in spherical geometry with smaller number of particles
- Easier to run simulations in large volume
- Fast redshift cone calculation
- Easier comparison with measurements (same methods can be used)

Future:

- Hubble-volume simulation
- Better IC generation